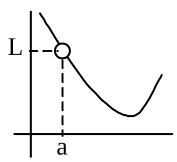
BAB XI
L I M I T

11. 1 Limit dan kontinuitas

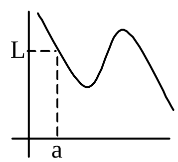
$\lim\limits_{x \to a} f(x) = L$ artinya nilai f(x) akan mendekati L untuk nilai x yang mendekati a.

Fungsi f(x) kontinu di x = a jika $\lim\limits_{x \to a} f(x) = f(a)$

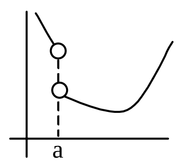
Berikut adalah sedikit ilustrasi tentang masalah limit dan kekontinuan suatu fungsi. Bisa kita lihat, nilai $\lim\limits_{x \to a} f(x)$ belum tentu sama dengan nilai f(a).



$\lim\limits_{x \to a} f(x) = L$
f(a) tidak terdefinisi
f(x) tidak kontinu di a



$\lim\limits_{x \to a} f(x) = L$
f(a) = L
f(x) kontinu di a



$\lim\limits_{x \to a} f(x)$ tidak ada
f(a) tidak terdefinisi
f(x) tidak kontinu di a

11. 2 Operasi pada limit

1. $\lim\limits_{x \to a} [\, f(x) + g(x) \,] = \lim\limits_{x \to a} f(x) + \lim\limits_{x \to a} g(x)$
2. $\lim\limits_{x \to a} [\, f(x) - g(x) \,] = \lim\limits_{x \to a} f(x) - \lim\limits_{x \to a} g(x)$
3. $\lim\limits_{x \to a} [\, C\ f(x) \,] = C \lim\limits_{x \to a} f(x)$, C suatu konstanta
4. $\lim\limits_{x \to a} [\, f(x) \,.\, g(x) \,] = \lim\limits_{x \to a} f(x) \,.\, \lim\limits_{x \to a} g(x)$
5. $\lim\limits_{x \to a} \dfrac{f(x)}{g(x)} = \dfrac{\lim\limits_{x \to a} f(x)}{\lim\limits_{x \to a} g(x)}$, dengan $\lim\limits_{x \to a} g(x) \neq 0$
6. $\lim\limits_{x \to a} [\, f(x) \,]^n = [\, \lim\limits_{x \to a} f(x) \,]^n$

11. 3. Limit fungsi Aljabar

Pada contoh berikut nilai $\lim\limits_{x \to a} f(x)$ dapat langsung dihitung, yaitu $\lim\limits_{x \to a} f(x) = f(a)$. Seperti telah dijelaskan sebelumnya, kita katakan fungsi f(x) kontinu di a .

Contoh :

1. $\lim\limits_{x \to 1} (x^3 + x) = 1^3 + 1 = 2$
2. $\lim\limits_{x \to 3} (2x^2 - x^3) = 2 \,.\, 3^2 - 3^3 = -9$

Pada bagian berikutnya, kita akan membahas penyelesaikan masalah limit fungsi aljabar bentuk tak tentu ($\frac{0}{0}, \frac{\infty}{\infty}, \infty - \infty, 0 \cdot \infty$)

11. 3. 1. Limit bentuk $\frac{0}{0}$

Bentuk $\lim\limits_{x\to a}\frac{f(x)}{g(x)}$ dimana f(a) = 0 dan g(a) = 0 desebut bentuk $\frac{0}{0}$. Pada bentuk ini f(x) dan g(x) akan mempunyai faktor yang sama (x –a). Limit bentuk ini diselesaikan dengan pencoretan faktor (x –a) yang sama tersebut.

Contoh :

1. $\lim\limits_{x\to -2}\frac{x^2-x-6}{x^2+9x+14} = \lim\limits_{x\to -2}\frac{(x+2)(x-3)}{(x+2)(x+7)} = \lim\limits_{x\to -2}\frac{x-3}{x+7} = \frac{-2-3}{-2+7} = -1$

2. $\lim\limits_{x\to 4}\sqrt{\frac{2x^2-7x-4}{3x^2-8x-16}} = \lim\limits_{x\to 4}\sqrt{\frac{2x^2-7x-4}{3x^2-8x-16}}$

$= \sqrt{\lim\limits_{x\to 4}\frac{(x-4)(2x+1)}{(x-4)(3x+4)}} = \sqrt{\lim\limits_{x\to 4}\frac{2x+1}{3x+4}} = \frac{3}{4}$

3. $\lim\limits_{x\to 2}\frac{\sqrt{2x^2+1}-\sqrt{5x-1}}{x^4-16} = \lim\limits_{x\to 2}\frac{\sqrt{2x^2+1}-\sqrt{5x-1}}{x^4-16}\cdot\frac{\sqrt{2x^2+1}+\sqrt{5x-1}}{\sqrt{2x^2+1}+\sqrt{5x-1}}$

$= \lim\limits_{x\to 2}\frac{2x^2-5x+2}{(x^2-4)(x^2+4)(\sqrt{2x^2+1}+\sqrt{5x-1})}$

$= \lim\limits_{x\to 2}\frac{(x-2)(2x-1)}{(x-2)(x+2)(x^2+4)(\sqrt{2x^2+1}+\sqrt{5x-1})}$

$= \lim\limits_{x\to 2}\frac{(2x-1)}{(x+2)(x^2+4)(\sqrt{2x^2+1}+\sqrt{5x-1})} = \frac{3}{4\cdot 8\cdot(3+3)} = \frac{1}{64}$

4. $\lim\limits_{x\to 1}\frac{x^2+bx-5}{x^2+7x-8} = k$, k suatu konstanta, maka b + k = …

Jawab :

Perhatikan Penyebut bentuk limit ini $1^2 + 7 . 1 - 8 = 0$.

Karena limit terdefinisi, maka bentuk limit adalah $\frac{0}{0}$

Dengan demikian $1^2 + b . 1 - 5 = 0 \Rightarrow b = 4$

Akibatnya $k = \lim\limits_{x\to 1}\frac{x^2+bx-5}{x^2+7x-8} = \lim\limits_{x\to 1}\frac{x^2+4x-5}{x^2+7x-8}$

$= \lim\limits_{x\to 1}\frac{(x-1)(x+5)}{(x-1)(x+8)} = \lim\limits_{x\to 1}\frac{x+5}{x+8} = \frac{6}{9} = \frac{2}{3}$

11. 3. 2. Limit bentuk $\frac{\infty}{\infty}$

Jika

$f(x) = a_0 x^n + a_1 x^{n-1} + \ldots + a_n$

$g(x) = b_0 x^n + b_1 x^{n-1} + \ldots + b_n$

Maka

$\lim\limits_{x\to\infty}\frac{f(x)}{g(x)} = \frac{a_0}{b_0}$ Untuk n = m

$\lim\limits_{x\to\infty}\frac{f(x)}{g(x)} = 0$ Untuk n < m

$\lim\limits_{x\to\infty}\frac{f(x)}{g(x)} = \infty$ Untuk n > m

Contoh

1. $\lim\limits_{x\to\infty} \dfrac{5x^4-3x^2+2x-1}{x^2+x-9} = \infty$

2. $\lim\limits_{x\to\infty} \dfrac{x^5+x^4-x^3+x^2+1}{10x^5+x^3-9} = \dfrac{1}{10}$

3. $\lim\limits_{x\to\infty} \dfrac{2x^3-x^2+12x-11}{x^5+x} = 0$

11. 3. 3. Limit bentuk ∞ – ∞

Bentuk umum : $\lim\limits_{x\to\infty} \sqrt{f(x)} - \sqrt{g(x)}$

Cara penyelesaian :

1. Kalikan dengan bentuk sekawannya (Baca : $\sqrt{f(x)} + \sqrt{g(x)}$)

$$\lim_{x\to\infty} \sqrt{f(x)} - \sqrt{g(x)} \cdot \frac{\sqrt{f(x)} + \sqrt{g(x)}}{\sqrt{f(x)} + \sqrt{g(x)}} = \lim_{x\to\infty} \frac{f(x) - g(x)}{\sqrt{f(x)} + \sqrt{g(x)}}$$

2. Bentuknya menjadi $\frac{\infty}{\infty}$, yang dapat diselesaikan dengan cara seperti bentuk 7.3.1

$\lim\limits_{x\to\infty} \sqrt{ax^2+bx+c} - \sqrt{px^2+qx+r} =$

1. $\dfrac{b-q}{2\sqrt{a}}$ untuk a = p

2. ∞ untuk a > p

3. –∞ untuk a < p

Contoh :

1. $\lim\limits_{x\to\infty} \sqrt{3x^2+x-2} - \sqrt{x^2+11x-7} = \infty$

2. $\lim\limits_{x\to\infty} \sqrt{x^2+2x-5} - \sqrt{5x^2+x-13} = -\infty$

3. $\lim\limits_{x\to\infty} \sqrt{x^2+11x-10} - \sqrt{x^2-4x+21} = \dfrac{11-(-4)}{2\sqrt{1}} = \dfrac{15}{2}$

4. $\lim\limits_{x\to\infty} (2x-3) - \sqrt{4x^2-5x+10} = \lim\limits_{x\to\infty} \sqrt{(2x-3)^2} - \sqrt{4x^2-5x+10}$

$= \lim\limits_{x\to\infty} \sqrt{4x^2-12x+9} - \sqrt{4x^2-5x+10}$

$= \dfrac{-12-(-5)}{2\sqrt{4}} = -\dfrac{7}{4}$

5. $\lim\limits_{x\to\infty} \sqrt{9x^2-2x-4} - (3x+2) = \lim\limits_{x\to\infty} \sqrt{9x^2-2x-4} - \sqrt{(3x+2)^2}$

$= \lim\limits_{x\to\infty} \sqrt{9x^2-2x-4} - \sqrt{9x^2+12x+4}$

$= \dfrac{-2-12}{2\sqrt{9}} = -\dfrac{7}{3}$

$$\lim_{x\to\infty} \sqrt[n]{p\,x^n+a_{n-1}x^{n-1}+...+a_0} - \sqrt[n]{p\,x^n+b_{n-1}x^{n-1}+...+b_0} = \frac{a_{n-1}-b_{n-1}}{n\sqrt[n]{p^{n-1}}}$$

Contoh :

1. $\lim\limits_{x\to\infty} \sqrt[3]{8x^3+2x^2+5} - \sqrt[3]{8x^3-3x^2+x-10} = \dfrac{2-(-3)}{3\sqrt[3]{8^2}} = \dfrac{5}{12}$

2. $\lim\limits_{x\to\infty} \sqrt[5]{243x^5-x^4+5x^3} - \sqrt[5]{243x^5-4x^4+x^2+3} = \dfrac{-1-(-4)}{5\sqrt[5]{(243)^4}} = \dfrac{1}{135}$

11. 4. Limit fungsi trigonometri

$$\lim_{x\to 0}\frac{\sin x}{x} = \lim_{x\to 0}\frac{x}{\sin x} = 1 \qquad \lim_{x\to 0}\frac{\tan x}{x} = \lim_{x\to 0}\frac{x}{\tan x} = 1$$

11. 4. 1. Limit bentuk $\frac{0}{0}$

1. $\lim\limits_{x\to 0}\dfrac{3x-\sin 4x+2x\tan x}{\sin^2 3x+\sin 5x} = \lim\limits_{x\to 0}\dfrac{x\left(3-\frac{\sin 4x}{x}+2\tan x\right)}{x\left(\frac{\sin 3x}{x}\sin 3x+\frac{\sin 5x}{x}\right)}$

$= \dfrac{3-4+2\cdot 0}{3\cdot 0+5} = -\dfrac{1}{5}$

2 $\lim\limits_{x\to 0}\dfrac{1-\cos^3 x}{\sin^2 x} = \lim\limits_{x\to 0}\dfrac{(1+\cos 2x+\cos^2 2x)(1-\cos 2x)}{\sin^2 x}$ — $a^3-b^3=(a^2+ab+b^2)(a-b)$

$= \lim\limits_{x\to 0}\dfrac{(1+\cos 2x+\cos^2 2x)(2\sin^2 x)}{\sin^2 x}$ — $\text{Cos } 2\alpha = 1-2\sin^2\alpha$

$= \lim\limits_{x\to 0} 2\,(1+\cos 2x+\cos^2 2x) = 2\,(1+1+1) = 6$

3. $\lim\limits_{x\to 0}\dfrac{1-\sqrt{\cos 4x}}{\tan 2x\cdot\sin x} = \lim\limits_{x\to 0}\dfrac{1-\sqrt{\cos 4x}}{\tan 2x\cdot\sin x}\cdot\dfrac{1+\sqrt{\cos 4x}}{1+\sqrt{\cos 4x}}$

$= \lim\limits_{x\to 0}\dfrac{1-\cos 4x}{(\tan 2x\cdot\sin x)(1+\sqrt{\cos 4x})}$

$= \lim\limits_{x\to 0}\dfrac{1-(1-2\sin^2 2x)}{(\tan 2x\,\sin x)(1+\sqrt{\cos 4x})}$ — $\text{Cos } 2\alpha = 1-2\sin^2\alpha$

$= \lim\limits_{x\to 0}\dfrac{2\sin^2 2x}{(\tan 2x\cdot\sin x)(1+\sqrt{\cos 4x})} = \dfrac{2\cdot(2)^2}{2\cdot 1\cdot(1+\sqrt{0})} = 4$

4. $\lim\limits_{x\to 0}\dfrac{\tan x-\sin x}{x-x\cos 4x} = \lim\limits_{x\to 0}\dfrac{\tan x-\tan x\cdot\cos x}{x-x\cos 4x} = \lim\limits_{x\to 0}\dfrac{\tan x\,(1-\cos x)}{x\,(1-\cos 4x)}$

$= \lim\limits_{x\to 0}\dfrac{\tan x\,(1-(1-2\sin^2\frac{1}{2}x))}{x\,(1-(1-2\sin^2 2x)} = \lim\limits_{x\to 0}\dfrac{2\tan x\cdot\sin^2\frac{1}{2}x}{2x\sin^2 2x}$

$= \dfrac{1}{16}$

5. $\lim\limits_{x\to\frac{\pi}{4}}\dfrac{\sin x-\cos x}{1-\tan x} = \lim\limits_{x\to\frac{\pi}{4}}\dfrac{\sin x-\cos x}{1-\frac{\sin x}{\cos x}} = \lim\limits_{x\to\frac{\pi}{4}}\dfrac{\sin x-\cos x}{1-\frac{\sin x}{\cos x}}\;\dfrac{\cos x}{\cos x}$

$= \lim\limits_{x\to\frac{\pi}{4}}\cos x\,\dfrac{\sin x-\cos x}{\cos x-\sin x} = \left(\frac{1}{2}\sqrt{2}\right)(-1) = -\frac{1}{2}\sqrt{2}$

11. 4. 2. Limit bentuk $\infty - \infty$ dan $0 \cdot \infty$

Limit trigonometri bentuk ini diselesaikan dengan mengubahnya ke bentuk $\frac{0}{0}$.

1. $\lim\limits_{x \to 0} \left(\frac{2}{x^2} - \frac{\sin 2x}{x^2 \text{ tgx}} \right) = \ldots$

Jawab

Limit bentuk diatas adalah $\infty - \infty$, ubah ke bentuk $\frac{0}{0}$

$$\lim_{x \to 0} \left(\frac{2}{x^2} - \frac{\sin 2x}{x^2 \text{ tgx}} \right) = \lim_{x \to 0} \frac{2\text{tgx} - \sin 2x}{x^2 \text{ tgx}} = \lim_{x \to 0} \frac{2\,\frac{\text{sinx}}{\text{cosx}} - 2\sin x \cos x}{x^2\,\frac{\text{sinx}}{\text{cosx}}}$$

$$= \lim_{x \to 0} \frac{2\sin x (1 - \cos^2 x)}{x^2 \sin x} = \lim_{x \to 0} \frac{2\sin x \sin^2 x}{x^2 \sin x} = 2$$

2. $\lim\limits_{x \to \frac{\pi}{2}} \tan x - \sec x = \ldots$

Jawab :

Limit bentuk diatas adalah $\infty - \infty$, ubah ke bentuk $\frac{0}{0}$

$$\lim_{x \to \frac{\pi}{2}} \tan x - \sec x = \lim_{x \to \frac{\pi}{2}} \frac{\sin x}{\cos x} - \frac{1}{\cos x} = \lim_{x \to \frac{\pi}{2}} \frac{\sin x - 1}{\cos x} \cdot \frac{\sin x + 1}{\sin x + 1}$$

$$= \lim_{x \to \frac{\pi}{2}} \frac{\sin^2 x - 1}{\cos x\,(\sin x + 1)} = \lim_{x \to \frac{\pi}{2}} \frac{-\cos^2 x}{\cos x\,(\sin x + 1)}$$

$$= \lim_{x \to \frac{\pi}{2}} \frac{-\cos x}{\sin x + 1} = \frac{-0}{1+1} = 0$$

3. $\lim\limits_{x \to \frac{3}{2}\pi} \left(x - \frac{3}{2}\pi \right) \sec x = \ldots$

(A) 1 **(B)** 2 **(C)** 3 **(D)** –1 **(E)** –2

Jawab :

Misalkan $t = x - \frac{3}{2}\pi$

$$\lim_{x \to \frac{3}{2}\pi} \left(x - \frac{3}{2}\pi \right) \sec x = \lim_{t \to 0} t \sec\left(t + \frac{3}{2}\pi\right) = \lim_{t \to 0} \frac{t}{\cos\left(t + \frac{3}{2}\pi\right)}$$

$$= \lim_{t \to 0} \frac{t}{\sin t} = 1$$

11. 5. Penyelesaian limit dengan D'Hospital

Jika $\lim\limits_{x \to a} \frac{f(x)}{g(x)}$ bentuk $\frac{0}{0}$, maka $\lim\limits_{x \to a} \frac{f(x)}{g(x)} = \lim\limits_{x \to a} \frac{f'(x)}{g'(x)}$

Contoh :

1. $$\lim_{x \to 1} \frac{x^4 + 3x^3 - 5x^2 + 1}{4x^5 + x^4 - 2x - 3} = \lim_{x \to 1} \frac{4x^3 + 9x^2 - 10x}{20x^4 + 4x^3 - 2} = \frac{4 + 9 - 10}{20 + 4 - 2} = \frac{3}{22}$$

2. $$\lim_{x \to 2} \frac{\sqrt[3]{x^2 + 2x} - \sqrt{3x - 2}}{x^5 - 32} = \lim_{x \to 2} \frac{\frac{1}{3}\,\frac{2x + 2}{\sqrt[3]{(x^2 + 3x - 2)^2}} - \frac{1}{2}\,\frac{3}{\sqrt{3x - 2}}}{5x^4}$$

$$= \frac{\frac{1}{3}\,\frac{6}{\sqrt[3]{8^2}} - \frac{1}{2}\,\frac{3}{\sqrt{4}}}{5 \cdot 16} = \frac{\frac{1}{2} - \frac{3}{4}}{80} = -\frac{1}{320}$$

3. $$\lim_{x \to \frac{1}{6}\pi} \frac{\cos\left(3x - \frac{1}{6}\pi\right) - \sin x}{\tan\left(x + \frac{1}{12}\pi\right) - 1} = \lim_{x \to \frac{1}{6}\pi} \frac{-3\sin\left(3x - \frac{1}{6}\pi\right) - \cos x}{\sec^2\left(x + \frac{1}{12}\pi\right)}$$

$$= \frac{-3\sin\left(\frac{1}{3}\pi\right) - \cos\left(\frac{1}{6}\pi\right)}{\sec^2\left(\frac{1}{4}\pi\right)} = \frac{-3\left(\frac{1}{2}\right) - \frac{1}{2}}{\left(\frac{1}{2}\right)^2} = -2$$

Soal dan Pembahasan Matematika Ipa

1. $\lim\limits_{x\to\infty} \sqrt{x^2+x+5}-\sqrt{x^2-2x+3} =$

   a. 0 b. $\frac{3}{2}$ c. $\sqrt{2}$ d. 2 e. ∞

   (Matematika '89 Rayon A)

   Jawab : B

   Ingat $\lim\limits_{x\to\infty} \sqrt{ax^2+bx+c} - \sqrt{px^2+qx+r} = \frac{b-q}{2\sqrt{a}}$, untuk a = p.

   Dengan demikian $\lim\limits_{x\to\infty} \sqrt{x^2+x+5}-\sqrt{x^2-2x+3} = \frac{1-(-2)}{2\sqrt{1}} = \frac{3}{2}$

2. Jika $\lim\limits_{x\to 0} \frac{\sin x}{x} = 1$ maka $\lim\limits_{x\to 0} \left(\frac{2}{x^2} - \frac{\sin 2x}{x^2 \mathrm{tg} x}\right) =$

   a. –2 b. –1 c. 0 d. 1 e. 2

   (Matematika '89 Rayon A)

   Jawab : E

   $$\lim_{x\to 0} \left(\frac{2}{x^2} - \frac{\sin 2x}{x^2 \mathrm{tg} x}\right) = \lim_{x\to 0} \left(\frac{2}{x^2} - \frac{2\sin x \cos x}{x^2} \cdot \frac{\cos x}{\sin x}\right)$$

   $$= \lim_{x\to 0} \frac{2}{x^2} - \frac{2\cos^2 x}{x^2} = \lim_{x\to 0} \frac{2-2\cos^2 x}{x^2}$$

   $$= \lim_{x\to 0} \frac{2(1-\cos^2 x)}{x^2} = \lim_{x\to 0} \frac{2\sin^2 x}{x^2} = 2$$

3. $\lim\limits_{x\to 3} \frac{1}{x-3}\left[\frac{1}{x-7} - \frac{2}{x-11}\right] =$

   a. $-\frac{1}{24}$ b. $-\frac{1}{32}$ c. 0 d. $\frac{1}{32}$ e. $\frac{1}{24}$

   (Matematika '89 Rayon B)

   Jawab : B

   $$\lim_{x\to 3} \frac{1}{x-3}\left[\frac{1}{x-7} - \frac{2}{x-11}\right] = \lim_{x\to 3} \frac{1}{x-3}\left[\frac{x-11-2(x-7)}{(x-7)(x-11)}\right]$$

   $$= \lim_{x\to 3} \frac{1}{x-3}\left[\frac{3-x}{(x-7)(x-11)}\right]$$

   $$= \lim_{x\to 3} \frac{-1}{(x-7)(x-11)} = \frac{-1}{(-4)(-8)} = -\frac{1}{32}$$

4. Jika $\lim\limits_{x\to 0} \frac{\sin x}{x} = 1$, maka $\lim\limits_{h\to 0} \frac{\sin(\frac{1}{3}\pi + h) - \sin\frac{1}{3}\pi}{h} =$

   a. $-\frac{1}{2}\sqrt{2}$ b. $-\frac{1}{2}$ c. $\frac{1}{2}$ d. $\frac{1}{2}\sqrt{2}$ e. $\frac{1}{2}\sqrt{3}$

   (Matematika '89 Rayon B)

   Jawab : C

Cara 1

$$\lim_{h\to 0}\frac{\sin(\frac{1}{3}\pi+h)-\sin\frac{1}{3}\pi}{h} = \lim_{h\to 0}\frac{2\sin[\frac{1}{2}(\frac{1}{3}\pi+h)]\ \cos[\frac{1}{2}(\frac{1}{3}\pi+h+\frac{1}{3}\pi)]}{h}$$

$$= \lim_{h\to 0}\frac{2\sin\frac{1}{2}h\cdot\cos(\frac{1}{2}h+\frac{1}{3}\pi)}{h}$$

$$= \lim_{h\to 0}\frac{2\sin\frac{1}{2}h}{h}\cdot\cos(\frac{1}{2}h+\frac{1}{3}\pi)$$

$$= 2\cdot\frac{1}{2}\cdot\cos(0+\tfrac{1}{3}\pi) = \cos\tfrac{1}{3}\pi = \frac{1}{2}$$

Cara 2

$f'(x) = \lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$.

Jika f(x) = sin x maka $\lim\limits_{h\to 0}\dfrac{\sin(\frac{1}{3}\pi+h)-\sin\frac{1}{3}\pi}{h} = f'(\frac{1}{3}\pi)$

Karena $f'(x) = \cos x$ maka $f'(\frac{1}{3}\pi) = \cos\frac{1}{3}\pi = \dfrac{1}{2}$

5. Jika $\lim\limits_{x\to 0}\dfrac{\sin x}{x} = 1$, maka $\lim\limits_{x\to 0}\dfrac{\sin 3x - \sin 3x\cos 2x}{4x^3} =$

a. $\frac{1}{2}$ b. $\frac{2}{3}$ c. $\frac{3}{4}$ d. $\frac{3}{2}$ e. 3

(Matematika '89 Rayon C)

Jawab : D

$$\lim_{x\to 0}\frac{\sin 3x-\sin 3x\ \cos 2x}{4x^3} = \lim_{x\to 0}\frac{\sin 3x}{4x}\left[\frac{1-\cos 2x}{x^2}\right]$$

$$= \lim_{x\to 0}\frac{\sin 3x}{4x}\left[\frac{1-(1-2\sin^2 x)}{x^2}\right]$$

$$= \lim_{x\to 0}\frac{\sin 3x}{4x}\left[\frac{2\sin^2 x}{x^2}\right] = \frac{3}{4}\cdot 2 = \frac{3}{2}$$

6. $\lim\limits_{x\to 0}\dfrac{4x}{\sqrt{1+2x}-\sqrt{1-2x}} =$

a. 0 b. 1 c. 2 d. 4 e. ∞

(Matematika '89 Rayon C)

Jawab : C

$$\lim_{x\to 0}\frac{4x}{\sqrt{1+2x}-\sqrt{1-2x}} = \lim_{x\to 0}\frac{4x}{\sqrt{1+2x}-\sqrt{1-2x}}\cdot\frac{\sqrt{1+2x}+\sqrt{1-2x}}{\sqrt{1+2x}+\sqrt{1-2x}}$$

$$= \lim_{x\to 0}\frac{4x(\sqrt{1+2x}+\sqrt{1-2x})}{(1+2x)-(1-2x)}$$

$$= \lim_{x\to 0}\frac{4x(\sqrt{1+2x}-\sqrt{1-2x})}{4x}$$

$$= \lim_{x\to 0}\sqrt{1+2x}+\sqrt{1-2x} = \sqrt{1+0}+\sqrt{1-0} = 2$$

Cara lain menggunakan D'Hospital

$$\lim_{x\to 0}\frac{4x}{\sqrt{1+2x}-\sqrt{1-2x}} = \lim_{x\to 0}\frac{4}{\frac{2}{2\sqrt{1+2x}}-\frac{-2}{2\sqrt{1-2x}}}$$

$$= \frac{4}{\frac{2}{2\sqrt{1+0}}-\frac{-2}{2\sqrt{1-0}}} = \frac{4}{1-(-1)} = 2$$

$f(x) = \sqrt{g(x)}$

$\Rightarrow f'(x) = \dfrac{g'(x)}{2\sqrt{g(x)}}$

7. $\lim\limits_{x\to 0} \frac{x \sin 3x}{1-\cos 4x} =$

a. $\frac{3}{8}$ b. $\frac{3}{4}$ c. $\frac{3}{2}$ d. $\frac{1}{4}$ e. $-\frac{3}{8}$

(Matematika ’90 Rayon A)

Jawab : A

$$\lim\limits_{x\to 0} \frac{x \sin 3x}{1-\cos 4x} = \lim\limits_{x\to 0} \frac{x \sin 3x}{1-(1-2\sin^2 2x)} = \lim\limits_{x\to 0} \frac{x \sin 3x}{2\sin^2 2x}$$

$$= \lim\limits_{x\to 0} \frac{x}{2\sin 2x} \cdot \frac{\sin 3x}{\sin 2x} = \frac{1}{2\cdot 2} \cdot \frac{3}{2} = \frac{3}{8}$$

8. $\lim\limits_{x\to 0} \frac{1-\cos x}{x \sin x} =$

a. 0 b. 1 c. 2 d. $\frac{1}{2}$ e. $\frac{1}{4}$

(Matematika ’90 Rayon B)

Jawab : D

$$\lim\limits_{x\to 0} \frac{1-\cos x}{x \sin x} = \lim\limits_{x\to 0} \frac{2\sin^2 \frac{1}{2}x}{x \sin x} = 2 \cdot \left(\frac{1}{2}\right)^2 = \frac{1}{2}$$

9. Jika $\lim\limits_{y\to 0} \frac{\sin y}{y} = 1$, maka $\lim\limits_{x\to 1} \frac{1-\cos^2 (x-1)}{4(x^2-2x+1)} =$

a. 0 b. $\frac{1}{4}$ c. $\frac{1}{2}$ d. 1 e. ∞

(Matematika ’90 Rayon C)

Jawab : B

Misal y = x – 1

$$\lim\limits_{x\to 1} \frac{1-\cos^2 (x-1)}{4(x^2-2x+1)} = \lim\limits_{x\to 1} \frac{\sin^2 (x-1)}{4(x-1)^2} = \lim\limits_{y\to 0} \frac{\sin^2 y}{4y^2} = \frac{1}{4}$$

10. $\lim\limits_{x\to 2} \frac{x-2}{3-\sqrt{x^2+5}} =$

a. $-\frac{3}{2}$ b. 0 c. $\frac{2}{3}$ d. $\frac{3}{2}$ e. 3

(Matematika ’91 Rayon A)

Jawab : A

Dengan mempergunakan D’Hospital

$$\lim\limits_{x\to 2} \frac{x-2}{3-\sqrt{x^2+5}} = \lim\limits_{x\to 2} \frac{1}{-\frac{2x}{2\sqrt{x^2+5}}}$$

$$= \lim\limits_{x\to 2} \frac{-2\sqrt{x^2+5}}{2x} = \frac{-2\sqrt{4+5}}{4} = -\frac{3}{2}$$

11. $\lim\limits_{h\to 0} \frac{\sqrt[3]{8+h}-2}{h} =$

a. ∞ b. 1 c. 0 d. $\frac{1}{12}$ e. $\frac{1}{8}$

(Matematika ’91 Rayon B)

Jawab : D

Pergunakan D’Hospital

$$\lim\limits_{h\to 0} \frac{(8+h)^{\frac{1}{3}}-2}{h} = \lim\limits_{h\to 0} \frac{\frac{1}{3}(8+h)^{-\frac{2}{3}}-0}{1} = \frac{1}{3}(8+0)^{-\frac{2}{3}} = \frac{1}{3}(2^3)^{-\frac{2}{3}}$$

$$= \frac{1}{3} \cdot 2^{-2} = \frac{1}{12}$$

12. $\lim\limits_{x \to 4} \dfrac{48 - 3x^2}{5 - \sqrt{x^2 + 9}} =$

a. 10 b. 20 c. 30 d. 40 e. 60

(Matematika ’91 Rayon C)

Jawab : C

Pergunakan D’Hospital

$$\lim_{x \to 4} \frac{48 - 3x^2}{5 - \sqrt{x^2 + 9}} = \lim_{x \to 4} \frac{0 - 6x}{0 - \dfrac{2x}{2\sqrt{x^2 + 9}}}$$

$f(x) = \sqrt{g(x)} \Rightarrow f'(x) = \dfrac{g'(x)}{2\sqrt{g(x)}}$

$$= \lim_{x \to 4} \frac{6x\,(2\sqrt{x^2 + 9})}{2x} = \frac{24 \cdot 2\sqrt{16 + 9}}{8} = 30$$

13. $\lim\limits_{x \to \infty} (3x - 2) - \sqrt{9x^2 - 2x + 5} =$

a. 0 b. $-\frac{1}{3}$ c. – 1 d. $-\frac{4}{3}$ e. $-\frac{5}{3}$

(Matematika ’92 Rayon A)

Jawab : E

Perhatikan $\lim\limits_{x \to \infty} (3x - 2) - \sqrt{9x^2 - 2x + 5} = \lim\limits_{x \to \infty} \sqrt{(3x-2)^2} - \sqrt{9x^2 - 2x + 5}$

$$= \lim_{x \to \infty} \sqrt{9x^2 - 12x + 4} - \sqrt{9x^2 - 2x + 5}$$

$\lim\limits_{x \to \infty} \sqrt{ax^2 + bx + c} - \sqrt{ax^2 + px + r} = \dfrac{b - p}{2\sqrt{a}}$

$$= \frac{-12 - (-2)}{2\sqrt{9}} = -\frac{10}{6} = -\frac{5}{3}$$

14. $\lim\limits_{x \to 0} \dfrac{x}{\sqrt{1 + x} - \sqrt{1 - x}} =$

a. 0 b. $\frac{1}{2}$ c. 1 d. $\sqrt{2}$ e. 4

(Matematika ’92 Rayon B)

Jawab : C

$$\lim_{x \to 0} \frac{x}{\sqrt{1 + x} - \sqrt{1 - x}} = \lim_{x \to 0} \frac{x}{\sqrt{1 + x} - \sqrt{1 - x}} \cdot \frac{\sqrt{1 + x} + \sqrt{1 - x}}{\sqrt{1 + x} + \sqrt{1 - x}}$$

$$= \lim_{x \to 0} \frac{x(\sqrt{1 + x} + \sqrt{1 - x})}{(1 + x) - (1 - x)} = \lim_{x \to 0} \frac{x(\sqrt{1 + x} + \sqrt{1 - x})}{2x}$$

$$= \lim_{x \to 0} \frac{\sqrt{1 + x} + \sqrt{1 - x}}{2} = \frac{\sqrt{1} + \sqrt{1}}{2} = \frac{2}{2} = 1$$

15. $\lim\limits_{x \to 1} \dfrac{\sqrt{x} - \sqrt{2x - 1}}{x - 1} =$

a. –1 b. $-\frac{1}{2}$ c. 0 d. $\frac{1}{2}$ e. 1

(Matematika ’92 Rayon C)

Jawab : B

$$\lim_{x \to 1} \frac{\sqrt{x} - \sqrt{2x - 1}}{x - 1} = \lim_{x \to 1} \frac{\sqrt{x} - \sqrt{2x - 1}}{x - 1} \cdot \frac{\sqrt{x} + \sqrt{2x - 1}}{\sqrt{x} + \sqrt{2x - 1}}$$

$$= \lim_{x \to 1} \frac{x - 2x + 1}{(x - 1)(\sqrt{x} + \sqrt{2x - 1})}$$

$$= \lim_{x \to 1} \frac{1 - x}{(x - 1)(\sqrt{x} + \sqrt{2x - 1})}$$

$$= \lim_{x \to 1} \frac{-1}{\sqrt{x} + \sqrt{2x - 1}} = \frac{-1}{\sqrt{1} + \sqrt{1}} = -\frac{1}{2}$$

16. Jika $\lim\limits_{x\to 4} \dfrac{ax + b - \sqrt{x}}{x - 4} = \dfrac{3}{4}$, maka a + b =

a. 3 b. 2 c. 1 d. –1 e. –2

(Matematika ’93 Rayon A, Rayon B, Rayon C)

Jawab : D

Limit tersebut merupakan limit tak tentu (untuk x = 4 nilai fungsinya $\frac{0}{0}$)

Untuk x = 4 pembilang = 0 $\Rightarrow$ $4a + b - \sqrt{4} = 0$ $\Rightarrow$ $4a + b = 2$
Pergunakan D’Hospital

$$\lim_{x\to 4} \frac{ax + b - \sqrt{x}}{x - 4} = \frac{3}{4} \Rightarrow \lim_{x\to 4} \frac{a + 0 - \frac{1}{2\sqrt{x}}}{1}$$

$$\Rightarrow a - \frac{1}{2\sqrt{4}} = \frac{3}{4} \Rightarrow a - \frac{1}{4} = \frac{3}{4} \Rightarrow a = 1$$

Karena 4a + b = 2 diperoleh b = –2. Jadi a + b = 1 + (–2) = –1

17. $\lim\limits_{x\to\infty} \left(\sqrt{(x + a)(x + b)} - x\right) =$

a. $\frac{a - b}{2}$ b. ∞ c. 0 d. $\frac{a + b}{2}$ e. a + b

(Matematika ’94 Rayon A)

Jawab : D

Perhatikan $\lim\limits_{x\to\infty} \left(\sqrt{(x + a)(x + b)} - x\right) = \lim\limits_{x\to\infty} \left(\sqrt{x^2 + (a+b)x + ab} - \sqrt{x^2}\right)$

$\lim\limits_{x\to\infty} \sqrt{ax^2 + bx + c} - \sqrt{ax^2 + px + r} = \dfrac{b - p}{2\sqrt{a}}$

$$= \frac{a + b - 0}{2\sqrt{1}} = \frac{a + b}{2}$$

18. $\lim\limits_{x\to\infty} \left(x - \sqrt{x^2 - 2x} \right) =$

a. ∞ b. 0 c. $\frac{1}{2}$ d. 1 e. 2

(Matematika ’94 Rayon B)

Jawab : D

$$\lim_{x\to\infty} \left(x - \sqrt{x^2 - 2x} \right) = \lim_{x\to\infty} \left(\sqrt{x^2} - \sqrt{x^2 - 2x} \right) = \frac{0 - (-2)}{2\sqrt{1}} = 1$$

19. Nilai dari $\lim\limits_{x\to\infty} \sqrt{x^2 - 5x} - x - 2$ adalah …

a. ∞ b. $\frac{1}{2}$ c. 0 d. $-\frac{9}{2}$ e. $-\frac{1}{2}$

(Matematika ’94 Rayon C)

Jawab : D

$$\lim_{x\to\infty} \sqrt{x^2 - 5x} - (x + 2) = \lim_{x\to\infty} \sqrt{x^2 - 5x} - \sqrt{(x + 2)^2}$$

$$= \lim_{x\to\infty} \sqrt{x^2 - 5x} - \sqrt{x^2 + 4x + 4} = \frac{-5 - 4}{2\sqrt{1}} = -\frac{9}{2}$$

20. $\lim\limits_{t\to 2} \dfrac{(t - 2)(t - 3)\ \sin(t - 2)}{[(t - 2)(t + 1)]^2} =$

a. $\frac{1}{3}$ b. $\frac{1}{9}$ c. 0 d. $-\frac{1}{9}$ e. $-\frac{1}{3}$

(Matematika ’95 Rayon A)

Jawab : D

$$\lim_{t\to 2} \frac{(t-2)(t-3)\ \sin(t-2)}{[(t-2)(t+1)]^2} = \lim_{t\to 2} \frac{(t-3)(t-2)\sin(t-2)}{(t+1)^2\,(t-2)^2}$$

$$= \lim_{t\to 2} \frac{(t-3)}{(t+1)^2} = \frac{2-3}{(2+1)^2} = -\frac{1}{9}$$

21. $\lim\limits_{x\to 0} \dfrac{(x^2-1)\sin 6x}{x^3+3x^2+2x} =$

a. –3 b. –1 c. 0 d. 1 e. 6

(Matematika ’95 Rayon B)

Jawab : A

$$\lim_{x\to 0} \frac{(x^2-1)\sin 6x}{(x^2+3x+2)x} = \lim_{x\to 0} \frac{x^2-1}{x^2+3x+2} \cdot \lim_{x\to 0} \frac{\sin 6x}{x} = \frac{0-1}{0+0+2} \cdot 6 = -3$$

22. $\lim\limits_{x\to -2} \dfrac{1-\cos(x+2)}{x^2+4x+4} =$

a. 0 b. $\frac{1}{4}$ c. $\frac{1}{2}$ d. 2 e. 4

(Matematika ’95 Rayon C)

Jawab : C

Ingat bahwa cos A = 1 – $2\sin^2 \frac{1}{2}$ A

misalkan x + 2 = y maka

Misal y = x + 2 $\Rightarrow \lim\limits_{x\to -2} \dfrac{1-\cos(x+2)}{x^2+4x+4} = \lim\limits_{x\to -2} \dfrac{1-\cos(x+2)}{(x+2)^2} = \lim\limits_{y\to 0} \dfrac{1-\cos y}{y^2}$

Ingat : cos A = 1 – $2\sin^2 \frac{1}{2}$ A

$$= \lim_{y\to 0} \frac{1-(1-2\sin\frac{1}{2}y)}{y^2}$$

$$= \lim_{y\to 0} \frac{2\sin\frac{1}{2}y}{y^2} = 2\left(\frac{1}{2}\right)^2 = \frac{1}{2}$$

23. $\lim\limits_{x\to 2} \left(\dfrac{2x^2-8}{x-2} + \dfrac{x^2-2x}{2x-4} \right) =$

a. 5 b. 6 c. 8 d. 9 e. ∞

(Matematika ’96 Rayon A)

Jawab : D

$$\lim_{x\to 2} \left(\frac{2x^2-8}{x-2} + \frac{x^2-2x}{2x-4}\right) = \lim_{x\to 2} \frac{2x^2-8}{x-2} + \lim_{x\to 2} \frac{x^2-2x}{2x-4}$$

$$= \lim_{x\to 2} \frac{4x-0}{1-0} + \lim_{x\to 2} \frac{2x-2}{2-0} = 8 + 1 = 9$$

24. $\lim\limits_{a\to b} \dfrac{a\sqrt{a}-b\sqrt{b}}{\sqrt{a}-\sqrt{b}} =$

a. 0 b. 3a c. $3\sqrt{b}$ d. 3b e. ∞

(Matematika ’96 Rayon B)

Jawab : D

Limit tersebut bervariabel a maka dengan D’Hospital

$$\lim_{a\to b} \frac{a\sqrt{a}-b\sqrt{b}}{\sqrt{a}-\sqrt{b}} = \lim_{a\to b} \frac{a^{\frac{3}{2}} - b\sqrt{b}}{a^{\frac{1}{2}} - \sqrt{b}} = \lim_{a\to b} \frac{\frac{3}{2}a^{\frac{1}{2}} - 0}{\frac{1}{2}a^{-\frac{1}{2}} - 0}$$

$$= \lim_{a\to b} 3 \cdot a^{\frac{1}{2}} \cdot a^{\frac{1}{2}} = \lim_{a\to b} 3a = 3b$$

25. $\lim\limits_{x\to\infty} \dfrac{\sqrt{2x^2+2x-3}-\sqrt{2x^2-2x-3}}{2} =$

a. 0 b. $\frac{1}{2}$ c. $\frac{1}{2}\sqrt{2}$ d. $\sqrt{2}$ e. ~

(Matematika '96 Rayon C)

Jawab : C

$$\lim_{x\to\infty} \frac{\sqrt{2x^2+2x-3}-\sqrt{2x^2-2x-3}}{2} = \frac{1}{2}\lim_{x\to\infty} \sqrt{2x^2+2x-3}-\sqrt{2x^2-2x-3}$$

$$= \frac{1}{2} \cdot \frac{2-(-2)}{2\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{1}{2}\sqrt{2}$$

26. $\lim\limits_{x\to 0} \dfrac{\sqrt{1+x}-1}{\sqrt[3]{1+x}-1} =$

a. 0 b. $\frac{1}{3}$ c. $\frac{2}{3}$ d. $\frac{3}{2}$ e. 2

(Matematika '97 Rayon A)

Jawab : D

$$\lim_{x\to 0} \frac{\sqrt{1+x}-1}{\sqrt[3]{1+x}-1} = \lim_{x\to 0} \frac{(1+x)^{\frac{1}{2}}-1}{(1+x)^{\frac{1}{3}}-1}$$

$$= \lim_{x\to 0} \frac{\frac{1}{2}(1+x)^{-\frac{1}{2}}-0}{\frac{1}{3}(1+x)^{-\frac{2}{3}}-0} = \frac{\frac{1}{2}(1+0)^{-\frac{1}{2}}}{\frac{1}{3}(1+0)^{-\frac{2}{3}}} = \frac{\frac{1}{2}}{\frac{1}{3}} = \frac{3}{2}$$

27. $\lim\limits_{x\to 3} \dfrac{x-\sqrt{2x+3}}{x^2-9} =$

a. $\frac{1}{3}$ b. $\frac{1}{9}$ c. $\frac{1}{6}$ d. $\frac{1}{2}$ e. 0

(Matematika '97 Rayon B)

Jawab : B

Pergunakan D'Hospital

$$\lim_{x\to 3} \frac{x-\sqrt{2x+3}}{x^2-9} = \lim_{x\to 3} \frac{1-\frac{1}{2\sqrt{2x+3}}}{2x}$$

$f(x) = \sqrt{g(x)} \Rightarrow f'(x) = \dfrac{g'(x)}{2\sqrt{g(x)}}$

$$= \frac{1-\frac{2}{2\sqrt{6+3}}}{6} = \frac{1-\frac{1}{3}}{6} = \frac{1}{9}$$

28. $\lim\limits_{x\to 1} \dfrac{\sqrt{x^2+3}-x-1}{1-x^2} =$

a $-\frac{1}{2}$ b. $-\frac{1}{4}$ c. 0 d. $\frac{1}{4}$ e. $\frac{1}{2}$

(Matematika '97 Rayon C)

Jawab : D

Pergunakan D'Hospital

$$\lim_{x\to 1} \frac{\sqrt{x^2+3}-x-1}{1-x^2} = \lim_{x\to 1} \frac{\frac{2x}{2\sqrt{x^2+3}}-1}{-2x} = \frac{\frac{2}{2\sqrt{4}}-1}{-2} = \frac{1}{4}$$

29. $\lim\limits_{x\to 1} \dfrac{\sqrt[3]{x^2}-2\sqrt[3]{x}+1}{(x-1)^2} = \ldots$

a. 0 b. $\frac{1}{3}$ c. $\frac{1}{5}$ d. $\frac{1}{7}$ e. $\frac{1}{9}$

(Matematika '98 Rayon A)

Jawab : B

$$\lim_{x\to 1}\frac{\sqrt[3]{x^2}-2\sqrt[3]{x}+1}{(x-1)^2} = \lim_{x\to 1}\frac{x^{\frac{2}{3}}-2x^{\frac{1}{3}}+1}{(x-1)^2}$$ Bentuk $\frac{0}{0}$ gunakan D'Hospital

$$= \lim_{x\to 1}\frac{\frac{2}{3}x^{-\frac{1}{3}}-\frac{2}{3}x^{-\frac{2}{3}}}{2(x-1)}$$ Bentuk $\frac{0}{0}$ gunakan D'Hospital

$$= \lim_{x\to 1}\frac{-\frac{2}{3}x^{-\frac{4}{3}}+\frac{4}{3}x^{-\frac{5}{3}}}{2} = \frac{-\frac{2}{3}+\frac{4}{3}}{2} = \frac{1}{3}$$

30. Nilai $\lim_{x\to 0}\frac{2x^2-5x}{3-\sqrt{9+x}}$ adalah …

a. 30 b. 1 c. 0 d. –1 e. –30

(Matematika '98 Rayon B)

Jawab : E

$$\lim_{x\to 0}\frac{2x^2-5x}{3-\sqrt{9+x}} = \lim_{x\to 0}\frac{2x^2-5x}{3-\sqrt{9+x}}\cdot\frac{3+\sqrt{9+x}}{3+\sqrt{9+x}}$$

$$= \lim_{x\to 0}\frac{(2x^2-5x)(3+\sqrt{9+x})}{9-(9-x)}$$

$$= \lim_{x\to 0}\frac{x(2x-5)(3+\sqrt{9+x})}{x}$$

$$= \lim_{x\to 0}(2x-5)(3+\sqrt{9+x}) = -30$$

31. $\lim_{x\to 1}\frac{x^{2n}-x}{1-x} =$

a. 2n – 1 b. 1 – 2n c. 2n d. 2n – 2 e. 2n + 2

(Matematika '98 Rayon C)

Jawab : B

Pergunakan D'Hospital

$$\lim_{x\to 1}\frac{x^{2n}-x}{1-x} = \lim_{x\to 1}\frac{2nx^{2n-1}-1}{-1} = \frac{2n\cdot(1)^{2n-1}-1}{-1} = \frac{2n-1}{-1} = 1-2n$$

Kumpulan soal Matematika Dasar

1. $\lim\limits_{x \to k} \dfrac{x-k}{\sin(x-k)+2k-2x} = \ldots.$

(A) -1 **(B)** 0 **(C)** $\frac{1}{3}$ **(D)** $\frac{1}{2}$ **(E)** 1

(UMPTN 99 RAYON A)

2. $\lim\limits_{x \to 1} \dfrac{1-\sqrt{x}}{1-x^2} = \ldots.$

(A) $-\frac{1}{2}$ **(B)** 0 **(C)** $\frac{1}{4}$ **(D)** 1 **(E)** 4

(UMPTN 99 RAYON A)

3. $\lim\limits_{x \to 0} \dfrac{\sin(x-2)}{x^2-4} = \ldots.$

(A) $-\frac{1}{4}$ **(B)** $-\frac{1}{2}$ **(C)** 0 **(D)** $\frac{1}{2}$ **(E)** $\frac{1}{4}$

(UMPTN 98 RAYON A)

4. $\lim\limits_{x \to 0} \dfrac{\sqrt{x}-x}{\sqrt{x}+x} = \ldots.$

(A) 0 **(B)** $\frac{1}{2}$ **(C)** 1 **(D)** 2 **(E)** ∞

(UMPTN 98 RAYON A)

5. Nilai $\lim\limits_{x \to 0} \left(\dfrac{\tan 2x \cdot \tan 3x}{5x^2}\right)$ adalah …

(A) 1 **(B)** $\frac{1}{5}$ **(C)** $\frac{2}{5}$ **(D)** $\frac{3}{5}$ **(E)** $\frac{6}{5}$

(UMPTN 98 RAYON B)

6. $\lim\limits_{x \to \infty} \dfrac{(4+5x)(2-x)}{(2+x)(1-x)} = \ldots$

(A) $-\infty$ **(B)** $\frac{1}{5}$ **(C)** 2 **(D)** 5 **(E)** ∞

(UMPTN 98 RAYON B)

7. $\lim\limits_{x \to 0} \dfrac{\sin 6x}{\sin 2x} = \ldots$

(A) $\frac{1}{6}$ **(B)** $\frac{1}{3}$ **(C)** 2 **(D)** 3 **(E)** 6

(UMPTN 98 RAYON C)

8. Nilai $\lim\limits_{x \to 2} \dfrac{x^3-8}{x^2-2x}$ adalah…

(A) 0 **(B)** 2 **(C)** 4 **(D)** 6 **(E)** ∞

(UMPTN 98 RAYON C)

9. $\lim\limits_{x \to 0} \dfrac{\tan x}{x^2+2x} =$

(A) 2 **(B)** 1 **(C)** 0 **(D)** $\frac{1}{2}$ **(E)** $\frac{1}{4}$

(UMPTN 97 RAYON A)

10. $\lim\limits_{t \to 4} \dfrac{\sqrt{t-2}}{t-4} =$

(A) 1 **(B)** $\frac{1}{4}$ **(C)** $\frac{1}{3}$ **(D)** $\frac{1}{2}$ **(E)** $\frac{3}{4}$

(UMPTN 97 RAYON A)

11. $\lim_{x \to 0} \frac{\sin x}{2x^2 - 2x} = \ldots$

(A) $-\frac{1}{2}$ **(B)** -1 **(C)** 0 **(D)** 1 **(E)** 2

(UMPTN 97 RAYON B)

12. $\lim_{x \to 7} \frac{x - 7}{\sqrt{x} - \sqrt{7}} =$

(A) $7\sqrt{7}$ **(B)** $3\sqrt{7}$ **(C)** $2\sqrt{7}$ **(D)** $\frac{1}{2\sqrt{7}}$ **(E)** $\frac{1}{\sqrt{7}}$

(UMPTN 97 RAYON B)

13. $\lim_{x \to 3} \frac{\sqrt{x} - \sqrt{3}}{x - 3} =$

(A) $\frac{1}{6}\sqrt{3}$ **(B)** $\frac{1}{3}\sqrt{3}$ **(C)** 1 **(D)** $\sqrt{3}$ **(E)** 3

(UMPTN 97 RAYON C)

14. $\lim_{x \to 0} \frac{2x^2 + x}{\sin x}$ adalah …

(A) 3 **(B)** 2 **(C)** 1 **(D)** 0 **(E)** -1

(UMPTN 97 RAYON C)